REPORT No. 589

AN ANALYSIS OF LATERAL STABILITY IN POWER-OFF FLIGHT WITH CHARTS FOR USE IN DESIGN

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SUMMARY

The aerodynamic and mass factors governing lateral stability are discussed and formulas are given for their estimation. Relatively simple relationships between the governing factors and the resulting stability characteristics are presented. A series of charts is included with which approximate stability characteristics may be rapidly estimated.

The effects of the various governing factors upon the stability characteristics are discussed in detail. It is pointed out that much additional research is necessary both to correlate stability characteristics with riding, flying, and handling qualities and to provide suitable data for accurate estimates of those characteristics of an airplane while it is in the design stage.

INTRODUCTION

The lateral stability of airplanes has been the subject of considerable mathematical treatment and many theoretical analyses. (See references.) The main aspects of the problem are therefore well known to students of the subject. Use of the mathematical theory in design is, however, limited by practical difficulties in its application. Determination of numerical values for certain of the aerodynamic quantities is difficult and the results are uncertain. The required calculations are extensive and must be carefully made to avoid erroneous and confusing results.

In this report lateral stability will be discussed and analyzed in a way that, it is believed, will aid in the acquisition of a working knowledge of the subject without long and intensive study. The classical equations have been simplified as much as seems consistent with reasonable accuracy to permit rapid estimation of the stability characteristics. Also included is a series of charts designed to facilitate the rapid estimation of the approximate lateral-stability characteristics of airplanes throughout the normal-flight range. It is hoped that these charts, together with those on longitudinal stability presented in reference 1, will aid in putting the estimation of the complete stability characteristics on a practical basis.

The material is presented in the following order: (1) A discussion of the aerodynamic and mass factors that govern the uncontrolled motion of the airplane together with formulas for estimating these factors; (2) formulas for estimating the stability characteristics of the uncontrolled motion having given the governing factors; (3) charts for the rapid estimation of stability characteristics; (4) a discussion of the effects of the governing factors upon the stability characteristics; (5) comments and suggestions for future study; (6) a brief derivation of the classical stability formulas (appendix I); (7) an accurate semigraphical method for solving biquadratics with a useful approximation based on this method (appendix II); and (8) a list of symbols and their definitions (appendix III).

FACTORS GOVERNING STABILITY

Both theory and experiment indicate that, with certain exceptions, the uncontrolled motion of an airplane can be divided into two independent phases. One phase includes components of the motion that do not displace the plane of symmetry of the airplane from the plane with which it coincides during the steady motion. Stability of this part of the motion is termed "longitudinal stability." The other phase of the complete motion includes all components that do displace the plane of symmetry. This phase of the motion is called "lateral motion" and its stability characteristics, "lateral stability." Although, in the past, reference has frequently been made to directional stability as distinguished from rolling stability (also called "lateral" stability), both theory and experiment indicate that no such division is physically possible for the conventional airplane.

The uncontrolled motion of an airplane quite obviously depends upon the aerodynamic forces and moments arising from any deviation from a steady state together with the inertial forces and moments accompanying the accelerations coupled with the deviations. The lateral motion is zero in steady flight on a straight course. The components of lateral motion in unsteady flight are a linear velocity v along the Y axis (see appendix I and fig. 1) and angular velocities p and r about the X and Z axes, respectively. The forces and moments governing lateral motion therefore arise from the aerodynamic reactions to the velocities v, p, and r (in the theoretical treatment aerodynamic reactions are assumed to be unaffected by accelerations) and the inertial reactions to the accelerations dv/dt, $g \sin \phi \cos \gamma$, $g \sin \psi \sin \gamma$, dp/dt, and dr/dt, where ϕ is the angle of roll, γ is the angle of the flight path, and ψ is the angle of yaw.

For convenience the components of the reactions referred to the coordinate axes are used rather than the resultant reaction. It appears, then, that a velocity v should result in a side force ΔY, a rolling moment ΔL, and a yawing moment ΔN. Similarly there will be Δ's of Y, L, and N corresponding to the rolling and yawing velocities p and r. The basis for the classical theory of stability is that the algebraic sum of the values of ΔY (for example) for a unit value of v when p and r are zero, for a unit value of p when v and r are zero, and for a unit value of r when v and p are zero is equal to the value of ΔY when the total motion is the resultant of coexisting unit values of v, p, and r. It is further assumed that a reaction ΔY due to a disturbance of velocity v is directly proportional to the magnitude of v, that is $\Delta Y = v\frac{dY}{dv}$. This assumption is admittedly an approximation but is valid, in general, for small values of the velocities of the disturbance. On this basis the aerodynamic reaction ΔY to a lateral disturbance is

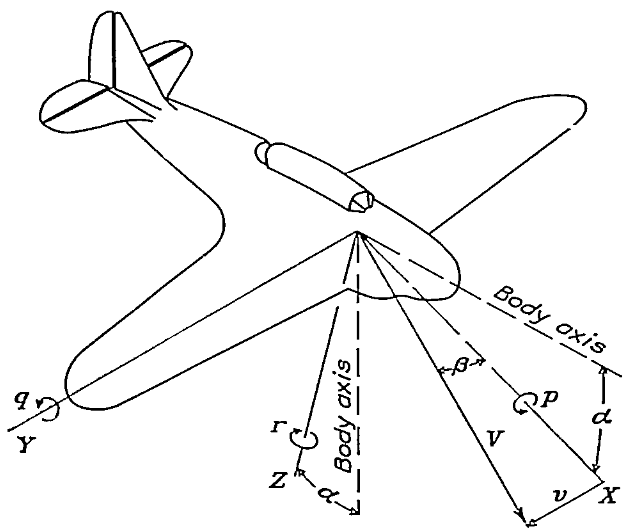


FIGURE 1.—Angular and vertical relationships in flight, power off.

$$\Delta Y = v\frac{dY}{dv} + p\frac{dY}{dp} + r\frac{dY}{dr}$$

and similar expressions exist for ΔL and ΔN.

As a matter of convenience it has been found desirable to express the derivatives dY/dv, dL/dv, etc., in terms of the nondimensional coefficients C_Y, C_l, and C_n where

$$C_Y = \frac{Y}{\frac{1}{2}\rho V^2 S}$$

$$C_l = \frac{L}{\frac{1}{2}\rho V^2 Sb}$$

$$C_n = \frac{N}{\frac{1}{2}\rho V^2 Sb}$$

In order to make the treatment entirely nondimensional, it is convenient to consider the ratios v/V, $pb/2V$, and $rb/2V$ rather than v, p, and r. For small values v/V is equal to β, where β is the angle of sideslip (in radians), and $pb/2V$ is the difference (in radians) between the angle of attack at the center of gravity and the angle of attack at the wing tip. Since the velocity at the wing tip is $V+rb/2$ the value $rb/2V$ is the ratio of the portion of the velocity at the tip due to rotation to the velocity at the center of gravity. Expressed in this way, the lateral-force coefficient due to lateral motion is

$$\Delta C_Y = \beta\frac{dC_Y}{d\beta} + \frac{pb}{2V}\frac{dC_Y}{d\frac{pb}{2V}} + \frac{rb}{2V}\frac{dC_Y}{d\frac{rb}{2V}}$$

and similar expressions exist for ΔC_l and ΔC_n.

Since $dC_Y/d\frac{pb}{2V}$ and $dC_Y/d\frac{rb}{2V}$ are small, they are generally neglected, leaving the following aerodynamic factors to be considered:

1. Those depending on sideslip: $dC_Y/d\beta$, $dC_l/d\beta$, and $dC_n/d\beta$.

2. Those depending on rolling velocity: $dC_l/d\frac{pb}{2V}$ and $dC_n/d\frac{pb}{2V}$.

3. Those depending on yawing velocity: $dC_l/d\frac{rb}{2V}$ and $dC_n/d\frac{rb}{2V}$

In addition to the aerodynamic factors, others that depend on the amount and the distribution of the mass of the airplane must be considered. The important mass factors, expressed nondimensionally, are μ, b/k_X, and b/k_Z. The relative density factor μ is equal to $m/\rho Sb$ and may be considered as being proportional to the ratio of the mass of the airplane to the mass of air influenced by it in traveling one chord length. Under standard conditions $\mu = \frac{13.1(W/S)}{b}$.

AERODYNAMIC FACTORS

Lateral force due to sideslip.—The rate of change of lateral-force coefficient with angle of sideslip $dC_Y/d\beta$ can be accurately determined only by measurement in a wind tunnel. Assuming the wind-tunnel data to have been obtained in terms of angle of yaw ψ in degrees, the value of $dC_Y/d\beta$ is -57.3 $(dC_Y/d\psi)$, since β is in radians and opposite in sign to ψ. In wind-tunnel practice, cross-wind force rather than lateral force is usually measured. In such cases $dC_Y/d\beta$ can be determined from the relationship

$$\frac{dC_Y}{d\beta} = \frac{dC_c}{d\beta} - C_D \tag{1}$$

(which follows from the fact that $C_Y=C_c \cos \beta - C_D \sin \beta$).

Diehl gives (reference 2, pp. 254–255) an approximate, empirical value of

$$\frac{dC_c}{d\beta}=-0.12\frac{bl_1}{S} \tag{2}$$

where l_1 is the over-all length. This formula is useful when wind-tunnel data are not available.

Rolling moment due to sideslip.—The rate of change of rolling-moment coefficient with sideslip $dC_l/d\beta$ must also be measured in a wind tunnel if accurate values are desired. Some systematic research has shown the effect of dihedral and tip shape on the value of $dC_l/d\beta$ for the wing alone (reference 3) but very little is known about the effect of fuselage interference. In certain experiments (data unpublished) a model having a wing with no dihedral mounted in a high-wing position gave a value of $dC_l/d\beta$ corresponding to 5° of positive dihedral for the wing alone. The same model with the wing mounted in a low-wing position gave a very erratic

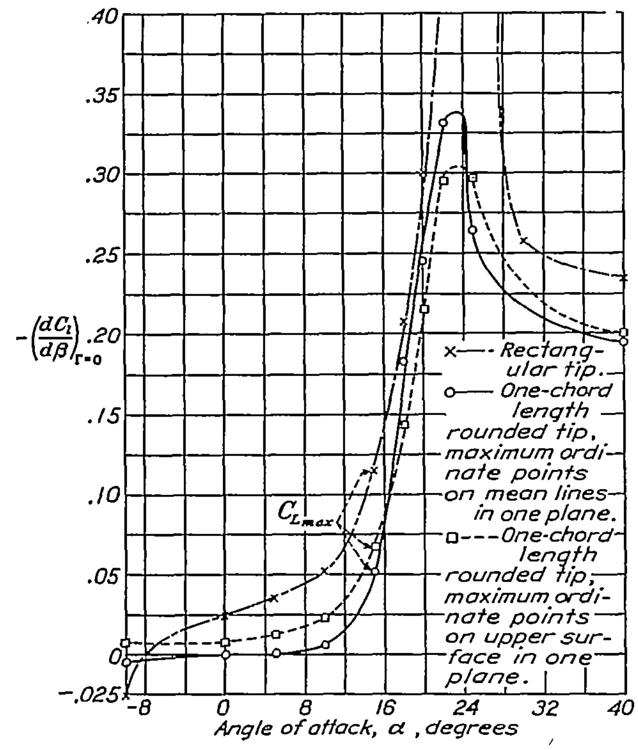


FIGURE 2.—Effect of tip shape on rate of change of rolling-moment coefficient with sideslip.

curve of C_l against β. The slope of this curve indicated zero dihedral effect at zero sideslip. The average dihedral effect up to 30° sideslip corresponded, however, to 4° negative dihedral. These tests were in the nature of preliminary tests and are unconfirmed but give ample evidence of the need for similar additional research.

In the absence of wind-tunnel tests the value of $dC_l/d\beta$ for the wing alone may be computed from the relationship

$$\frac{dC_l}{d\beta}=\left(\frac{dC_l}{d\beta}\right)_{\Gamma=0}+\Gamma(-0.012) \tag{3}$$

where $(dC_l/d\beta)_{\Gamma=0}$ is the value of $dC_l/d\beta$ for the wing without dihedral (see fig. 2) and Γ is the dihedral angle in degrees. This formula was developed from data obtained with wings of aspect ratio 6 and with no taper or sweepback (reference 3). Tapering the wing decreases the effective dihedral but the decrease is somewhat less than would be expected from the geometric proportions because of the tendency of the wing lift to be evenly distributed along the span. Sweepback is equivalent to an increase in dihedral, particularly at high angles of attack, but the effect is negligible for small amounts of sweepback such as are used in conventional airplanes.

The wing, including interference effects, is the chief source of rolling moment due to sideslip and other parts of the airplane can normally be neglected. Vertical-fin area displaced from the longitudinal axis contributes to $dC_l/d\beta$ but the effect is usually small. If, in a particular case, the effect upon the value of $dC_l/d\beta$ is desired for parts having considerable projected side area, it can be computed from the relationship

$$\frac{dC_l}{d\beta}=\frac{S_p}{S}\frac{z_p}{b}\frac{dC_{L_p}}{d\beta} \tag{4}$$

where S_p is the projected side area.

S, the wing area.

z_p, the z coordinate of the center of pressure of projected side area.

C_{L_p}, the absolute coefficient of force on the projected side area.

In this equation $dC_{L_p}/d\beta$ must be estimated, taking into account the shape of the part and the probable interference effects.

Yawing moment due to sideslip.—The change of yawing-moment coefficient with angle of sideslip $dC_n/d\beta$ depends principally upon the fuselage and the vertical-tail area. The contributions of the landing gear, interference effects, etc., are small and can generally be neglected. The contribution of the wings is also small and can be neglected at high or cruising speeds but becomes of increasing importance at slower speeds (reference 3); the effect due to the wing is an increase in $dC_n/d\beta$. The center of pressure upon the fuselage is normally well ahead of the center of gravity so that the moment due to sideslip is such as to increase the sideslip. The magnitude of this unstabilizing tendency varies with the size and shape of the fuselage but, on the average, is equal to about one-third the stabilizing effect of the vertical tail surfaces.

For accurate stability calculations it is necessary that $dC_n/d\beta$ be obtained from wind-tunnel tests at several angles of attack by the use of the relation $dC_n/d\beta=-57.3\ dC_n/d\psi$. The value of $dC_n/d\beta$ can be calculated approximately from the relation

$$\frac{dC_n}{d\beta}=\eta_t\frac{l}{b}\frac{S_t}{S}\left(\frac{dC_{L_t}}{d\beta}\right)-K_\beta\frac{S_s l_2}{Sb} \qquad (5)$$

where η_t is the tail efficiency.

l, the distance from the center of gravity to the rudder hinge.

l_2, the over-all length of the fuselage.

S_t, the area of the vertical tail surfaces.

C_{L_t}, the absolute coefficient of force on the vertical tail surfaces.

K_β, an empirical constant (reference 2, p. 203).

S_s, the projected side area of the fuselage.

When using equation (5), it is necessary to estimate or assume values of η_t, $dC_{L_t}/d\beta$, and K_β. For modern types of airplane η_t is about 0.80. The slope of the tail-force curve $dC_{L_t}/d\beta$ depends on the aspect ratio of the tail and to a certain extent upon the end-plate effect of the fuselage and the horizontal surfaces. For

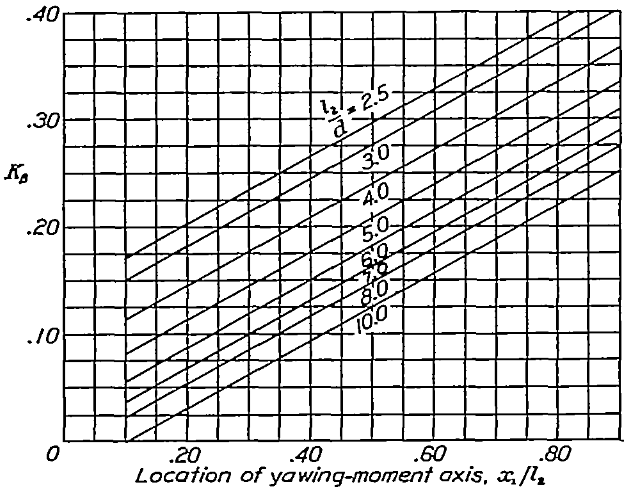


FIGURE 3.—Empirical factor for computing effect of fuselage on rate of change of yawing-moment coefficient with sideslip (from fig. 98, reference 2).

$$K_\beta=\frac{Sb}{S_s l_2}\left(\frac{dC_n}{d\beta}\right)_{\text{fuselage}}$$

conventional arrangements $dC_{L_t}/d\beta=2.2$ is a good average value. Values of K_β as determined by Diehl are given in figure 3. From this figure the value of K_β can be directly determined from the ratio of the distance of the center of gravity back of the nose x_1 to the fuselage length l_2 and the ratio of the maximum fuselage depth d to the fuselage length. In a number of computations made to check the accuracy of formula (5) it was found that the results were generally conservative, i. e., the estimated value of $dC_n/d\beta$ was smaller than the measured value. The difference arose in most cases from the fact that the measured effect of the fuselage was smaller than the estimated effect. The measured effect of the fuselage apparently varies between zero and the effect calculated as $K_\beta\frac{S_s l_2}{Sb}$ depending upon the details of nose shape and fuselage form.

Rolling moment due to rolling.—The rate of change of rolling-moment coefficient with rate of rolling $dC_l/d\frac{pb}{2V}$ arises from the change of angle of attack along the wing. The increment in angle of attack at any spanwise distance y from the center of gravity is py/V (in radian measure), the increment at the tip being $pb/2V$. If a uniform spanwise distribution of lift and drag be assumed, simple integration gives

$$\frac{dC_l}{d\frac{pb}{2V}}=\frac{-(dC_L/d\alpha+C_{D_w})}{6}$$

where C_{D_w} is the drag coefficient of the wing alone. If an elliptical distribution is assumed, integration gives

$$\frac{dC_l}{d\frac{pb}{2V}}=\frac{-(dC_L/d\alpha+C_{D_w})}{8}$$

Actually, measured values of $dC_l/d\frac{pb}{2V}$ are considerably smaller than either formula indicates because of the tendency of the lift to equalize itself along the span during the rotation.

In the absence of data obtained from some such device as a rolling balance, $dC_l/d\frac{pb}{2V}$ can be taken as -0.40 for wing arrangements such as are likely to be used on conventional airplanes. A survey of test results reveals values from -0.35 to -0.47 for plain wings and values as high as -0.50 for wings with tip slots. It would be expected that rounding the tips or tapering the wings would reduce $dC_l/d\frac{pb}{2V}$, and such was found to be the case for the tests reported in reference 3. On the other hand, there is sufficient conflicting evidence to indicate that an attempt to calculate $dC_l/d\frac{pb}{2V}$ taking into account tip shape and taper, is likely to give a result no nearer the true value than is the assumed average, -0.40.

Yawing moment due to rolling.—The rate of change of yawing-moment coefficient with rate of rolling $dC_n/d\frac{pb}{2V}$ arises from the same causes as does $dC_l/d\frac{pb}{2V}$. Simple integration gives for a rectangular-wing force distribution

$$\frac{dC_n}{d\frac{pb}{2V}}=\frac{-\left[C_L-\left(\frac{dC_{D_w}}{d\alpha}\right)\right]}{6}$$

and for an elliptical-wing force distribution gives

$$\frac{dC_n}{d\frac{pb}{2V}}=\frac{-\left[C_L-\left(\frac{dC_{D_w}}{d\alpha}\right)\right]}{8}$$

It will be noticed that the sign of the resulting value of $dC_n/d\frac{pb}{2V}$ indicates that the wing being depressed by the rolling motion is accelerated forward by the resulting yawing moment. The mistake has frequently been made (see reference 4) of assuming that the increase in drag of the wing being depressed would result in a yawing moment retarding that wing, that is, in a positive value of $dC_n/d\frac{pb}{2V}$. This reasoning fails to take into account the forward inclination with increase in angle of attack of the resultant-force vector relative to an axis fixed in the wing.

Wind-tunnel data cannot ordinarily be obtained for $dC_n/d\frac{pb}{2V}$ because there are but few existing balances capable of measuring the yawing moment on a rolling model. It is therefore necessary to rely on estimated values of $dC_n/d\frac{pb}{2V}$. The empirical relationship

$$\frac{dC_n}{d\frac{pb}{2V}}=\frac{-\left(C_L-1.1\frac{dC_{D_w}}{d\alpha}\right)}{8} \tag{6}$$

has been found (reference 5) to give good agreement with measured values below the stall, C_L and $dC_{D_w}/d\alpha$ having been obtained from force tests of the wing alone; but there is need for further experimental data on this factor.

Rolling moment due to yawing.—The rate of change of rolling-moment coefficient with rate of yawing $dC_l/d\frac{rb}{2V}$ results from the difference in velocity between the wing tips, one wing tip having the velocity $V+rb/2$ and the other having the velocity $V-rb/2$. Simple integration gives the value of $dC_l/d\frac{rb}{2V}$ as $C_L/3$ assuming a rectangular distribution of lift or as $C_L/4$ assuming an elliptical distribution. The rolling moment due to yawing is of positive sign since a positive rate of yawing gives a positive rolling moment.

It will ordinarily be impossible to measure $dC_l/d\frac{rb}{2V}$ for a particular design because of lack of equipment. Either special apparatus for oscillating the model or a whirling arm equipped to measure rolling moments is required. In the absence of experimental data, the computed value must be used. Glauert states (reference 6) that

$$\frac{dC_l}{d\frac{rb}{2V}}=\frac{C_L}{4} \tag{7}$$

gives nearly correct values for a rectangular wing. Experimental results for the Bristol Fighter, a biplane with substantially rectangular wings, however, gave $dC_l/d\frac{rb}{2V}$ as nearly $C_L/3$ (reference 7). Measured values from tests of a biplane model reported in reference 8 were approximately equal to $C_L/4$ for three wing combinations. It appears that the assumption that equation (7) gives reasonable values is justified for wings with faired or elliptical tips and slight to moderate taper.

Yawing moment due to yawing.—The rate of change of yawing-moment coefficient due to yawing $dC_n/d\frac{rb}{2V}$ results from the change of velocity along the wing and the change of sideslip velocity along the fuselage and at the tail due to the yawing. On the basis of simple integration the portion of $dC_n/d\frac{rb}{2V}$ due to the wing is $-C_{D_w}/3$ for a rectangular distribution and $-C_{D_w}/4$ for an elliptical distribution. In an extension of work by Wieselsberger, Glauert shows (reference 6) that $dC_n/d\frac{rb}{2V}$ is equal to $-(0.33\ C_{D_0}+0.043\ C_{D_i})$ for a rectangular wing of normal aspect ratio and equal to $-(0.25\ C_{D_0}+0.33\ C_{D_i})$ for an elliptical wing, where C_{D_0} and C_{D_i} are the profile and induced drags, respectively, for the wing alone.

The change in angle of sideslip at the tail due to a yawing velocity r is rl/V. The theoretical value for the vertical tail is

$$\frac{dC_n}{d\frac{rb}{2V}}=-2\eta_t\frac{l^2}{b^2}\frac{S_t}{S}\frac{dC_{L_t}}{d\beta}$$

It will be noted that both the wing and the tail contributions to $dC_n/d\frac{rb}{2V}$ are negative; that is, they are in the sense to oppose the rotation.

It is unfortunate that experimental means of measuring $dC_n/d\frac{rb}{2V}$ are not more commonly available. As will appear later in the report, an accurate knowledge of $dC_n/d\frac{rb}{2V}$ is necessary for reasonable accuracy in estimating stability characteristics. The sparse experimental evidence concerning the value of this factor (see references 7 and 8) indicates that there are, in some cases, large interference effects. For one model tested on a whirling arm the value of $dC_n/d\frac{rb}{2V}$ for the fuselage and tail surfaces combined was only one-third the value for the tail surfaces alone. It is quite evident from such data that computed values can be considered at best only as rough approximations. With this limitation in mind it appears that the most suitable formula for $dC_n/d\frac{rb}{2V}$ is

$$\frac{dC_n}{d\frac{rb}{2V}}=-\frac{C_{D_w}}{3}-2\eta_t\frac{l^2}{b^2}\frac{S_t}{S}\frac{dC_{L_t}}{d\beta} \tag{8}$$

and that there is no justification for refinements in the formula.

MASS FACTORS

Relative density of airplane and air.—The relative density of the airplane to the air is usually expressed as $\mu=m/\rho Sb$. From this definition μ is $\pi/4$ times the ratio of the mass of the airplane to the mass of air affected by a monoplane wing (on the basis of accepted wing theory) in traveling a distance equal to the mean chord. It thus appears that μ is intimately tied up with the performance characteristics of the airplane.

For standard conditions $\mu=\frac{13.1(W/S)}{b}$. For other conditions μ may be expressed as

$$\frac{13.1(W/S)}{b}\frac{\rho_0}{\rho} \tag{9}$$

where ρ_0 is the standard mass density (0.002378 slug per cubic foot) and ρ is the actual mass density. It appears that μ increases with wing loading and altitude and decreases with span. The numerical value of μ ranges from 2 for large transports to 10 for pursuit airplanes under standard conditions. It appears that large airplanes are dynamically similar to very lightly loaded small airplanes, a transport with a span of 120 feet and wing loading of 25 corresponding to an airplane of 30-foot span with a wing loading of 6.25.

Ratio of wing span to radius of gyration about X axis.—The ratio of the wing span to the radius of gyration about the X axis, b/k_X, has been determined for 15 airplanes (reference 9) and has been found to range from 6.7 to 9.3, with 8.0 as an average value. This ratio can be estimated with sufficient accuracy for stability calculations. For preliminary estimates the average value of 8.0 is satisfactory for conventional types because, as will appear later, stability characteristics are not critically dependent upon the mass distribution.

Ratio of wing span to radius of gyration about Z axis.—The value of the ratio of the wing span to the radius of gyration about the Z axis, b/k_Z, has been found to vary from 5.1 to 6.4, with 5.7 as an average value. As in the case of b/k_X, the average value is satisfactory for most estimates of stability. The value of b/k_Z can be estimated with sufficient accuracy for all stability calculations from a weight analysis of the airplane.

STABILITY DERIVATIVES

In practice it has been found convenient to combine the aerodynamic and mass factors that govern lateral-stability characteristics into stability derivatives. These derivatives take the following forms, one for each of the aerodynamic factors:

$$\begin{aligned}
y_v&=\frac{1}{2}\frac{dC_Y}{d\beta}\\
l_v&=\frac{1}{2}\left(\frac{b}{k_X}\right)^2\frac{dC_l}{d\beta}\\
n_v&=\frac{1}{2}\left(\frac{b}{k_Z}\right)^2\frac{dC_n}{d\beta}\\
l_p&=\frac{1}{4}\left(\frac{b}{k_X}\right)^2\frac{dC_l}{d\frac{pb}{2V}}\\
n_p&=\frac{1}{4}\left(\frac{b}{k_Z}\right)^2\frac{dC_n}{d\frac{pb}{2V}}\\
l_r&=\frac{1}{4}\left(\frac{b}{k_X}\right)^2\frac{dC_l}{d\frac{rb}{2V}}\\
n_r&=\frac{1}{4}\left(\frac{b}{k_Z}\right)^2\frac{dC_n}{d\frac{rb}{2V}}
\end{aligned}$$

Physically these derivatives are, respectively, proportional to the linear or angular acceleration arising from a unit angle of sideslip, a unit rolling velocity as expressed by $pb/2V$, or a unit yawing velocity as expressed by $rb/2V$.

The stability derivatives include all the important factors governing stability characteristics except μ. Since μ occurs only in combination with n_v and l_v and, conversely, since these derivatives occur only in combination with μ, the lateral-stability characteristics can be completely expressed in terms of the seven nondimensional quantities: y_v, μl_v, μn_v, l_p, n_p, l_r, and n_r.

For preliminary estimates it will generally be sufficiently accurate to use the following values for the stability derivatives:

$$\left.\begin{aligned}
y_v&=-0.14\\
l_v&=32\frac{dC_l}{d\beta}\\
n_v&=16\frac{dC_n}{d\beta}\\
l_p&=-6.4\\
n_p&=-0.5\ C_L\\
l_r&=4\ C_L\\
n_r&=8\frac{dC_n}{d\frac{rb}{2V}}
\end{aligned}\right\} \tag{10}$$

A rather small value of y_v has been chosen in order to be conservative. Stability characteristics calculated with this small value of y_v can be readily corrected to correspond to a different y_v, a fact which will be subsequently shown. The derivatives l_p, n_p, and l_r may differ considerably from the foregoing values, particularly at angles of attack above that at which the lift-curve slope begins to decrease. Fortunately the stability characteristics are not greatly affected by moderate variation in these particular factors. If possible, values of $dC_l/d\beta$, $dC_n/d\beta$, and $dC_n/d\frac{rb}{2V}$ should be obtained by actual measurement. There is strong reason for believing that, unless these factors are accurately measured, a false impression of the accuracy of the estimated stability characteristics may be obtained by refinements in estimating the other factors.

FORMULAS FOR ESTIMATING STABILITY CHARACTERISTICS

Stability characteristics about which information is desired.—The preceding portion of this paper has dealt with the various aerodynamic and mass factors that govern stability characteristics. In the following paragraphs these factors will be grouped in relationships which show the effects of the individual factors upon the stability characteristics and from which these characteristics can be quantitatively determined.

Instability can manifest itself either as a continuously increasing divergence from the steady-flight condition or as an oscillation of continuously increasing amplitude about the steady-flight condition. On a logical basis it appears that the questions answered by an estimation of stability characteristics should be: (1) Will there exist a tendency to diverge from the steady-flight condition? (2) Will the oscillations started by a disturbance or by the use of the controls damp out and, if so, how quickly? (3) What will be the period of the lateral oscillations? Approximate relationships to answer these questions have been developed (see appendixes I and II) and are presented in the following pages.

Basis for formulas.—The following formulas are based on the classical theory of small oscillations first applied to airplane dynamics by Bryan and developed and expanded by Bairstow, Wilson, Glauert, and others (references 10 to 13). A brief derivation of the formulas is given in appendix I. The formulas presented represent a first approximation to a semigraphical method of accurately solving the stability biquadratic given by the classical theory. This semigraphical method and the approximation to it are explained in appendix II.

Formulas for predicting a divergence.—Divergence is not possible in the normal-flight range (to which this report is confined) if

$$\mu l_v n_r > \mu n_v l_r \tag{11}$$

and

$$\mu l_v\left(n_p-\frac{C_L}{2}\right)-\mu n_v l_p>0 \tag{12}$$

Failure to meet the first of these conditions results in "spiral divergence," a form of divergence in which the airplane tends to go into a spiral dive. Failure to meet the second condition results in "directional divergence," in which the airplane tends to yaw away from the direction of steady flight.

For purposes of approximate estimation using the values for the derivatives given in equation (10), equations (11) and (12) become

$$4(dC_l/d\beta)\left(dC_n/d\frac{rb}{2V}\right)>C_L(dC_n/d\beta) \tag{13}$$

and

$$-C_L(dC_l/d\beta)+3.2(dC_n/d\beta)>0 \tag{14}$$

If the contributions of the wings, the fuselage, and interference effects upon $dC_n/d\frac{rb}{2V}$ and upon $dC_n/d\beta$ are neglected, equation (13) further simplifies to

$$-dC_l/d\beta>\frac{C_L}{8}\frac{b}{l} \tag{15}$$

This latter equation is, however, an oversimplification for any but the most approximate analyses.

Formulas for estimating the damping of an oscillation.—The number of seconds required for an oscillation to damp to one-half its original amplitude is

$$T=\frac{-0.313\sqrt{\left(\frac{W}{S}\right)\frac{\rho_0}{\rho}C_L}}{\zeta'} \tag{16}$$

where ζ' is the damping coefficient. The time to damp to any other proportion of the original amplitude is given by

$$T_n=T\frac{\log_e n}{-0.693} \tag{17}$$

where n is the desired proportion, such as ¼ or ⅛. To a fairly close approximation (±15 percent)

$$\zeta'=-\frac{1}{2}\left[-y_v+\frac{l_pn_r-l_rn_p}{(-n_r-l_p)}+\frac{\mu n_v(-n_r)}{(-n_r-l_p)^2}+\frac{C_Ll_rn_v}{2l_v\left(n_p-\frac{C_L}{2}\right)-2n_vl_p}\right]+\frac{1}{2}\left[\frac{\mu l_v\left(n_p-\frac{C_L}{2}\right)}{(-n_r-l_p)^2}+\frac{C_Ll_vn_r}{2l_v\left(n_p-\frac{C_L}{2}\right)-2n_vl_p}\right] \tag{18}$$

In equation (18) the terms in the first pair of brackets are those which make ζ' more negative, i. e., decrease the time required for the oscillation to damp; the terms in the second pair of brackets are those which make ζ' less negative.

If the values from equations (10) are used, equation (18) becomes

$$\zeta' = -0.07 - \frac{25.6\left(-\frac{dC_n}{d\frac{rb}{2V}}\right)+C_L^2}{6.4+8\left(-\frac{dC_n}{d\frac{rb}{2V}}\right)} + \frac{16\ C_L\mu\left(-\frac{dC_l}{d\beta}\right)-64\ \mu\frac{dC_n}{d\beta}\left(\frac{dC_n}{d\frac{rb}{2V}}\right)}{\left[6.4+8\left(-\frac{dC_n}{d\frac{rb}{2V}}\right)\right]^2} + \frac{4\left(-\frac{dC_l}{d\beta}\right)\left(-\frac{dC_n}{d\frac{rb}{2V}}\right)-C_L\frac{dC_n}{d\beta}}{2\left(-\frac{dC_l}{d\beta}\right)+\frac{6.4}{C_L}\left(\frac{dC_n}{d\beta}\right)} \tag{19}$$

Since $8\left(-\frac{dC_n}{d\frac{rb}{2V}}\right)$ is small compared with 6.4 a further simplification is obtained by letting $8\left(-\frac{dC_n}{d\frac{rb}{2V}}\right)$, where it appears in the denominator, equal 0.84. On this basis

$$\zeta' = -0.07-3.5\left(\frac{dC_n}{d\frac{rb}{2V}}\right)-0.14\ C_L^2-1.2\ \mu\frac{dC_n}{d\beta}\left(-\frac{dC_n}{d\frac{rb}{2V}}\right) + 0.3\ \mu\left(-\frac{dC_l}{d\beta}\right)+\frac{4\left(-\frac{dC_l}{d\beta}\right)\left(-\frac{dC_n}{d\frac{rb}{2V}}\right)-C_L\frac{dC_n}{d\beta}}{2\left(-\frac{dC_l}{d\beta}\right)+\frac{6.4}{C_L}\frac{dC_n}{d\beta}} \tag{20}$$

Formula (20) will lead to fairly large errors if the airplane departs very far from the average. The error is roughly on a percentage basis so that, for small values of damping approaching an undesirable condition, the actual error is small.

Formulas for estimating the period of an oscillation.—The period, in seconds, of the lateral oscillation is

$$P = \frac{2.83\sqrt{\left(\frac{W}{S}\right)\frac{\rho_0}{\rho}C_L}}{\psi'} \tag{21}$$

where ψ' is the period coefficient. To a fairly close approximation ψ' is given by

$$\psi' = \sqrt{\frac{\mu l_r\left(n_p-\frac{C_L}{2}\right)-\mu n_r l_p}{(-n_r-l_p)}} \tag{22}$$

substituting the values for the derivatives given in equations (10) and letting $8\left(-dC_n/d\frac{rb}{2V}\right)=0.84$ gives

$$P = \sqrt{\frac{0.14b}{\left(-\frac{dC_l}{d\beta}\right)+\frac{3.2}{C_L}\frac{dC_n}{d\beta}}} \tag{23}$$

which is correct for the conventional airplane within ±20 percent.

CHARTS FOR ESTIMATING STABILITY CHARACTERISTICS

Explanation of charts.—A series of 22 charts for use in rapid estimation of stability characteristics are given in figures 4 to 25. In these charts the damping and the period of lateral oscillations are given by curves of $T/\sqrt{W/S}$=constant and of $P/\sqrt{W/S}$=constant plotted with $-\mu dC_l/d\beta$ as abscissas and $\mu dC_n/d\beta$ as ordinates. The limits to the region within which both spiral and static directional stability exist are indicated by straight lines representing zero spiral stability and zero directional stability, respectively. The rates of convergence or divergence are not given.

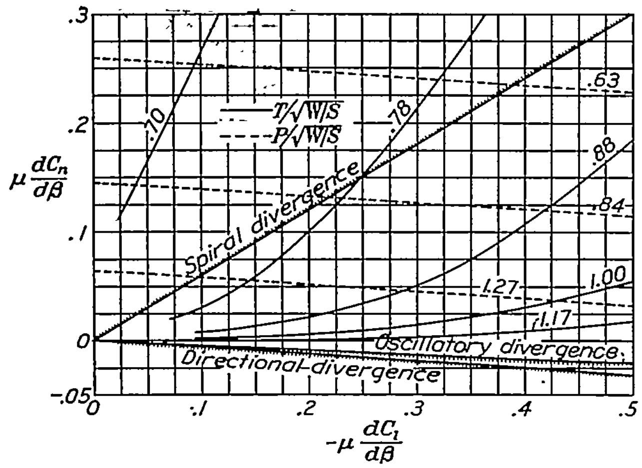
The charts cover values of C_L(0.2 to 2.0) and $dC_n/d\frac{rb}{2V}$ (−0.030 to −0.252) likely to occur in practice with conventional airplanes.

Each chart covers values of $-\mu dC_l/d\beta$ from 0 to 0.5 and of $\mu dC_n/d\beta$ from −0.05 to 0.3. These ranges are sufficiently large for most conventional airplanes. Some extrapolation is permissible in particular cases without much loss of accuracy other than that due to the fundamental weakness of increasing inaccuracy as the damping becomes large.

These charts are based on equations (13), (14), (19), and (23) and are therefore approximations to the same extent as the equations. They are intended principally for use in rapid estimates in design and show fairly accurately the relative effects of changes in C_L, $dC_n/d\frac{rb}{2V}$, $\mu dC_l/d\beta$, and $\mu dC_n/d\beta$. Being based on average values of $dC_Y/d\beta$, $dC_l/d\frac{pb}{2V}$, $dC_n/d\frac{pb}{2V}$, $dC_l/d\frac{rb}{2V}$, b/k_X, and b/k_Z, they cannot be used to determine the effect of changes in these factors. The charts should not be used where very accurate values are desired. On the other hand, there is little justification for using a more accurate method unless measured values of the various aerodynamic factors are available. If $dC_Y/d\beta$ is known to be much larger than −0.28, as, for example, in the case of an airplane with a split flap at a high angle of attack, correction for the damping can be made by the procedure given in the following section.

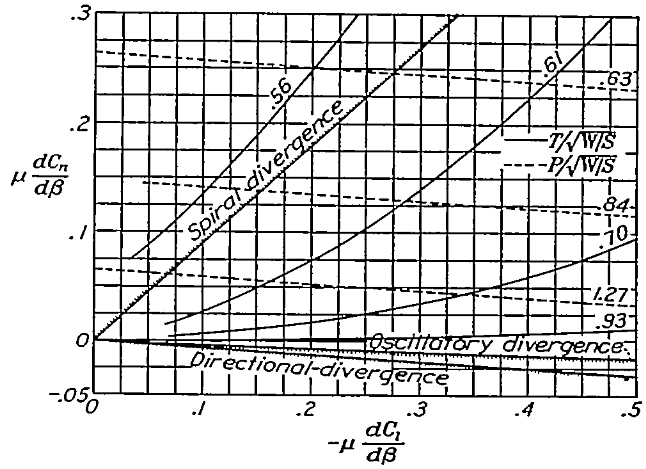
Method of using charts.—In order to use the charts the following data are needed:

W/S, wing loading.
b, wing span.
C_L, lift coefficient.
$dC_l/d\beta$, rate of change of rolling-moment coefficient with sideslip, per radian.
$dC_n/d\beta$, rate of change of yawing-moment coefficient with sideslip, per radian.
$dC_n/d\frac{rb}{2V}$, rate of change of yawing-moment coefficient with rate of yawing, per unit of $rb/2V$.



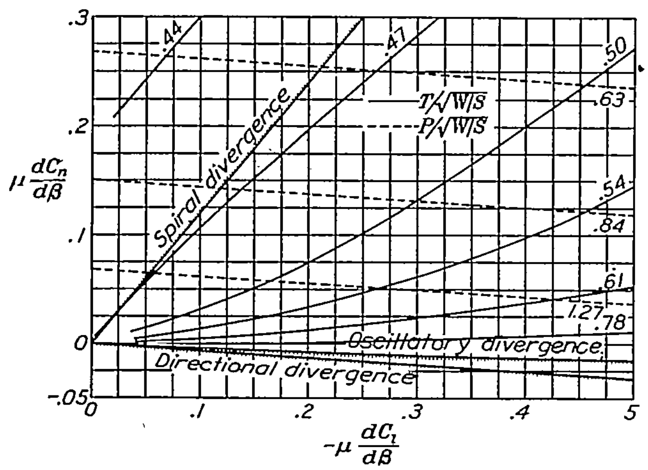
$C_L=0.2 \quad dC_n/d\frac{rb}{2V}=-0.030$

FIGURE 4.



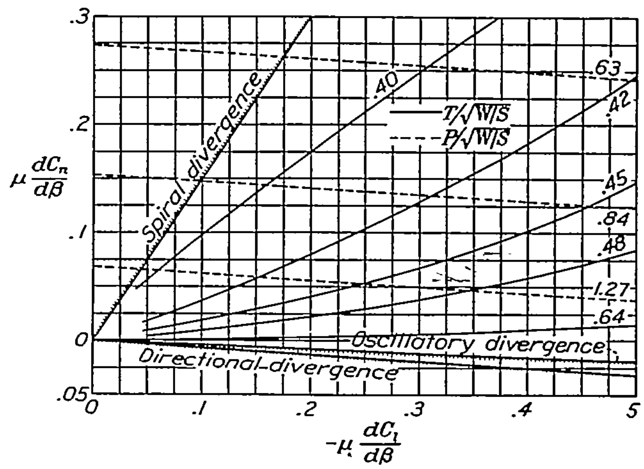
$C_L=0.2 \quad dC_n/d\frac{rb}{2V}=-0.045$

FIGURE 5.



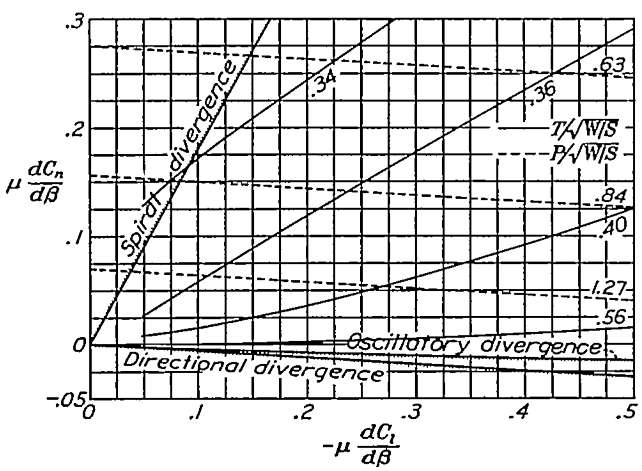
$C_L=0.2 \quad dC_n/d\frac{rb}{2V}=-0.060$

FIGURE 6.



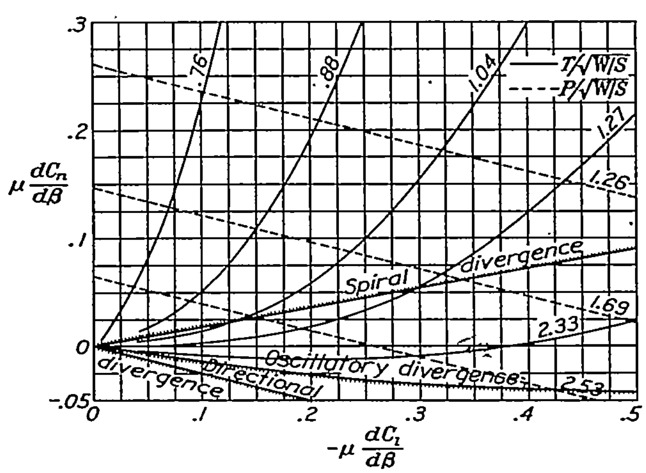
$C_L=0.2 \quad dC_n/d\frac{rb}{2V}=-0.075$

FIGURE 7.



$C_L=0.2 \quad dC_n/d\frac{rb}{2V}=-0.090$

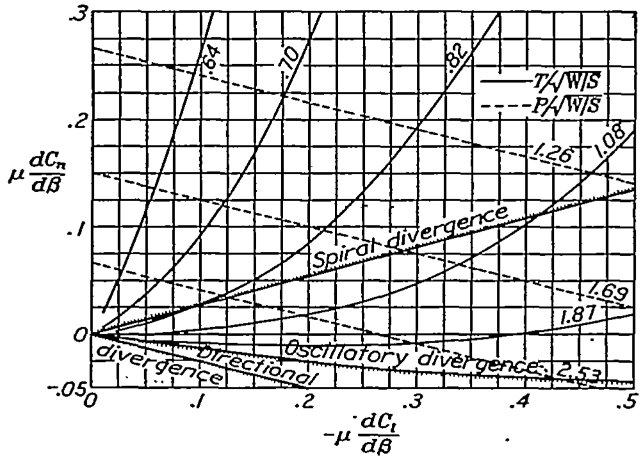
FIGURE 8.



$C_L=0.8 \quad dC_n/d\frac{rb}{2V}=-0.036$

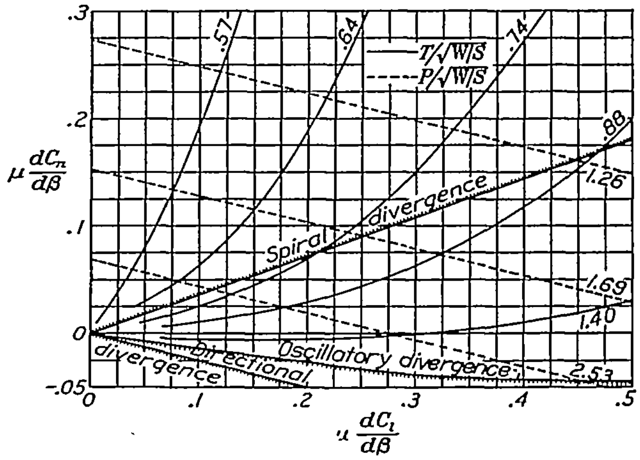
FIGURE 9.

FIGURES 4 to 9.—Lateral-stability charts.



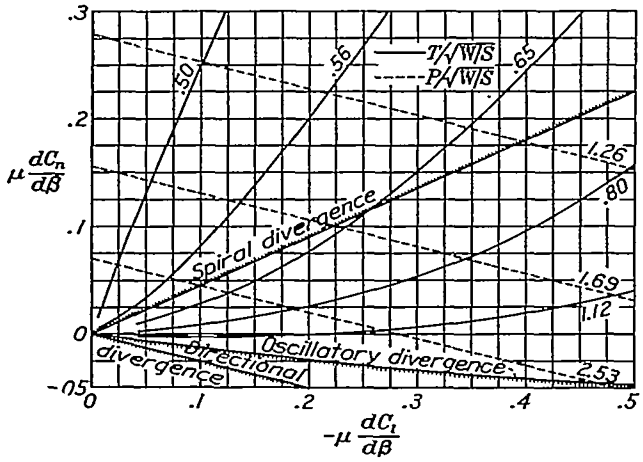
$C_L=0.8 \quad dC_n/d\frac{rb}{2V}=-0.054$

FIGURE 10.



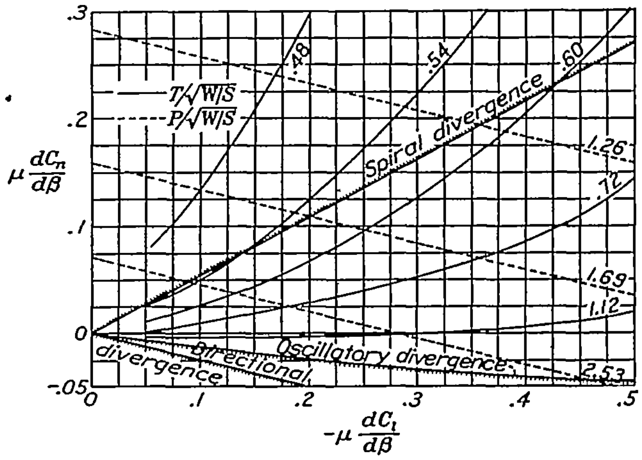
$C_L=0.8 \quad dC_n/d\frac{rb}{2V}=-0.072$

FIGURE 11.



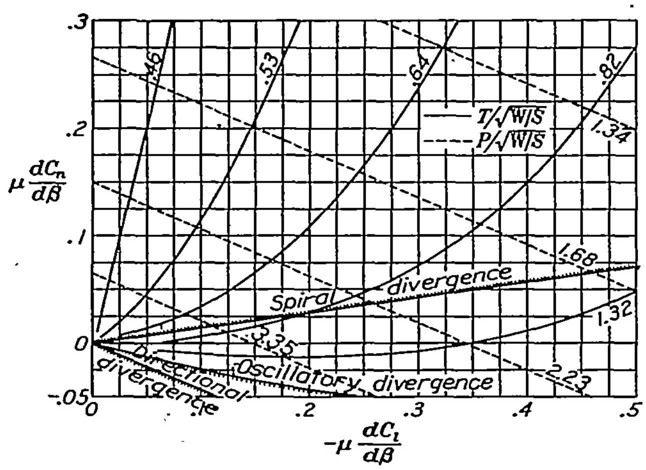
$C_L=0.8 \quad dC_n/d\frac{rb}{2V}=-0.090$

FIGURE 12.



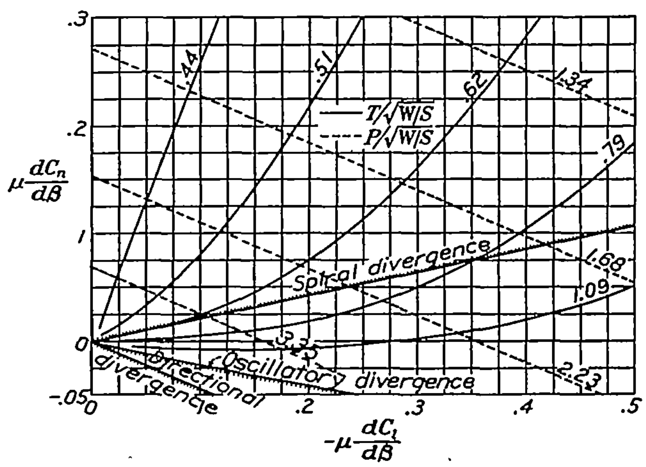
$C_L=0.8 \quad dC_n/d\frac{rb}{2V}=-0.108$

FIGURE 13.



$C_L=1.4 \quad dC_n/d\frac{rb}{2V}=-0.050$

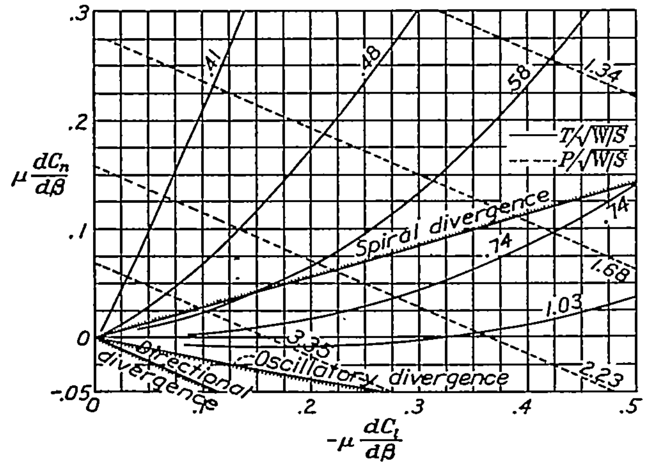
FIGURE 14.



$C_L=1.4 \quad dC_n/d\frac{rb}{2V}=-0.075$

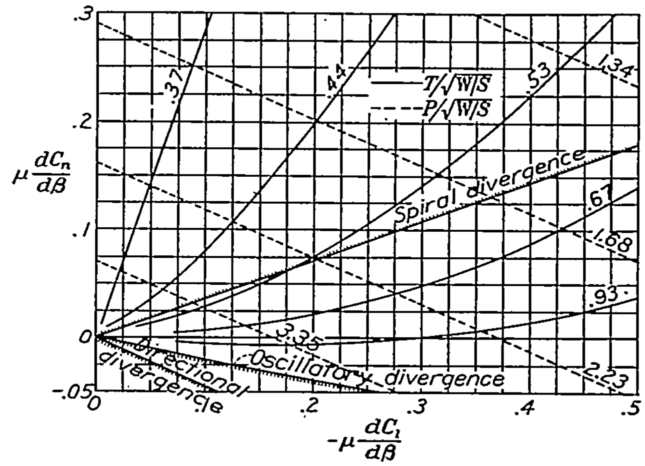
FIGURE 15.

FIGURES 10 to 15.—Lateral-stability charts.



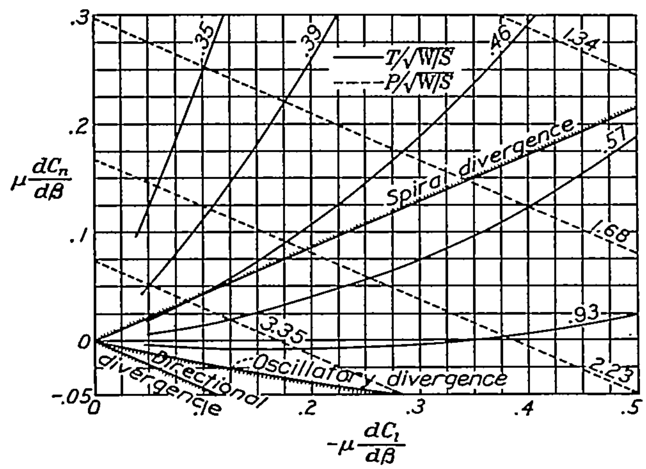
$C_L=1.4 \quad dC_n/d\frac{rb}{2V}=-0.100$

FIGURE 16.



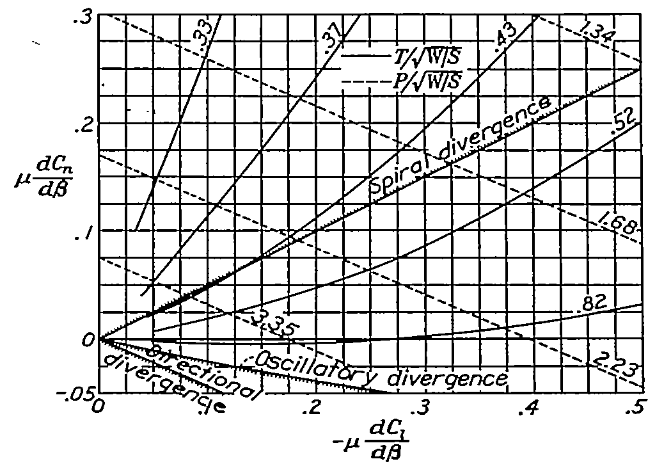
$C_L=1.4 \quad dC_n/d\frac{rb}{2V}=-0.125$

FIGURE 17.



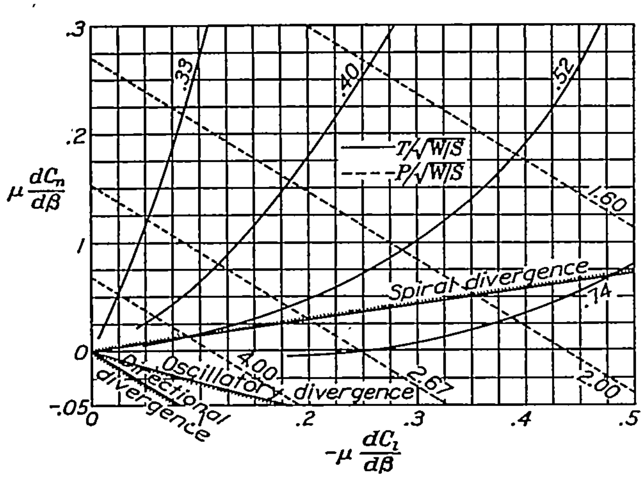
$C_L=1.4 \quad dC_n/d\frac{rb}{2V}=-0.150$

FIGURE 18.



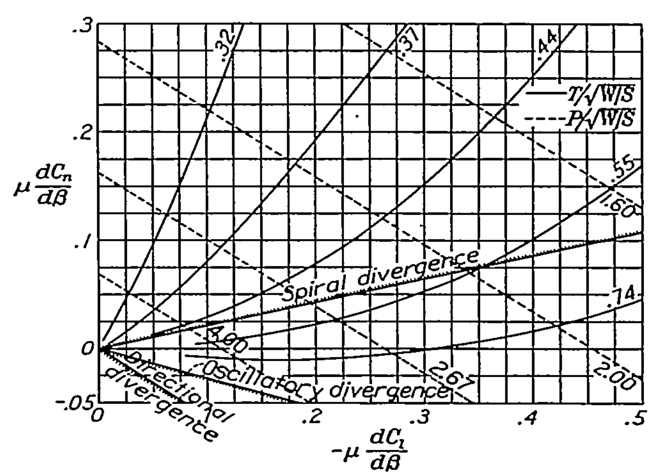
$C_L=1.4 \quad dC_n/d\frac{rb}{2V}=-0.175$

FIGURE 19.



$C_L=2.0 \quad dC_n/d\frac{rb}{2V}=-0.072$

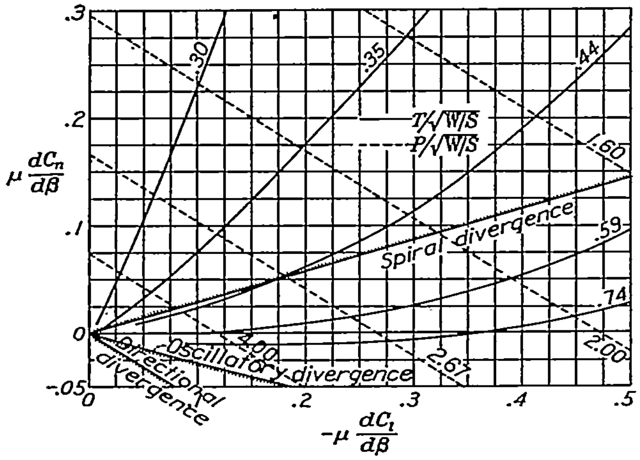
FIGURE 20.



$C_L=2.0 \quad dC_n/d\frac{rb}{2V}=-0.108$

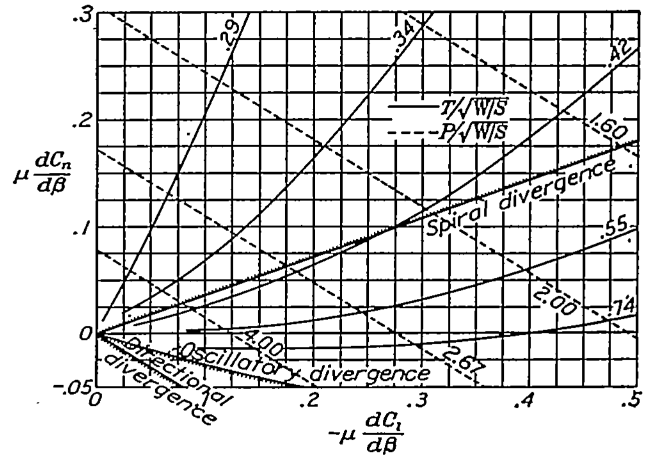
FIGURE 21.

FIGURES 16 to 21.—Lateral-stability charts.



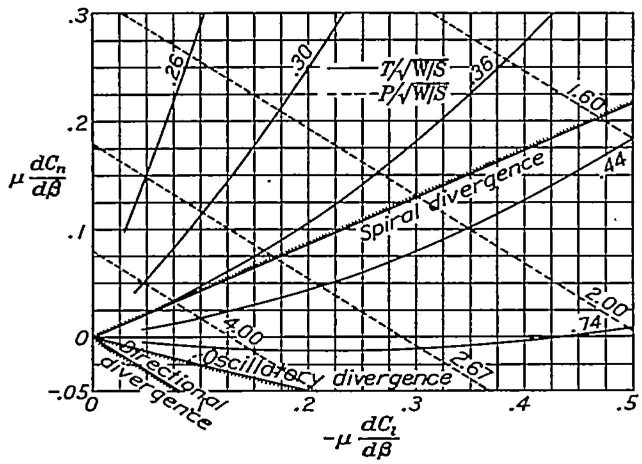
$C_L=2.0$ $dC_n/d\frac{rb}{2V}=-0.144$

FIGURE 22.



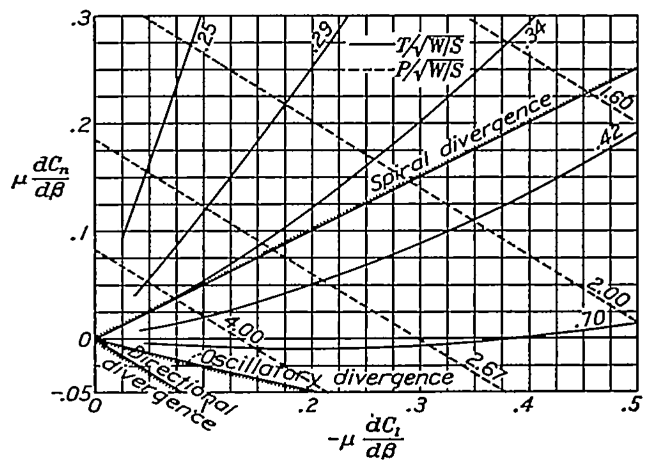
$C_L=2.0$ $dC_n/d\frac{rb}{2V}=-0.180$

FIGURE 23.



$C_L=2.0$ $dC_n/d\frac{rb}{2V}=-0.216$

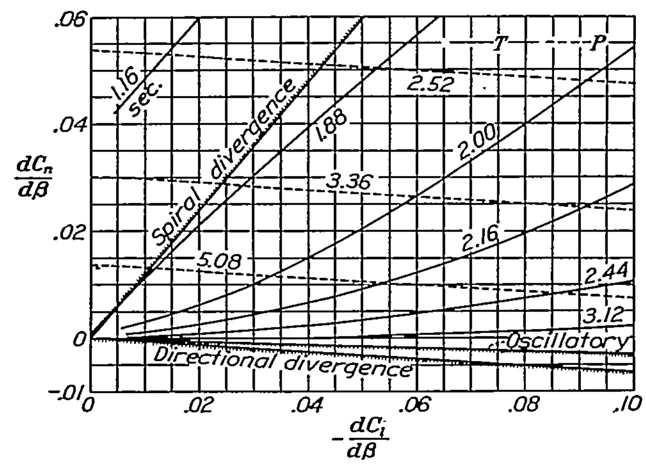
FIGURE 24.



$C_L=2.0$ $dC_n/d\frac{rb}{2V}=-0.252$

FIGURE 25.

FIGURES 22 to 25.—Lateral-stability charts.



$W/S=16$ lb./sq. ft.
$b=42$ ft. $C_L=0.2$
$dC_n/d\frac{rb}{2V}=-0.060$

FIGURE 26.—Special case of lateral-stability chart

From these data, values of $-\mu dC_l/d\beta$ and $\mu dC_n/d\beta$ can be readily determined since $\mu=\frac{13.1(W/S)}{b}$ (standard conditions).

In general, the value of C_L to represent a particular range of flight conditions can be chosen as 0.2, 0.8, 1.4, or 2.0. It will be necessary in most cases, however, to interpolate between two charts for the value of $dC_n/d\frac{rb}{2V}$. Any point given by $-\mu dC_l/d\beta$, $\mu dC_n/d\beta$ represents a value of $P/\sqrt{W/S}$ and a value of $T/\sqrt{W/S}$. The period and the time to damp to one-half amplitude are readily obtained by multiplying these values by $\sqrt{W/S}$. The location of the point $-\mu dC_l/d\beta$, $\mu dC_n/d\beta$ also indicates whether there will be a tendency to diverge.

The charts are computed for standard conditions. They can be easily applied to a study of stability at altitude by substituting the value $(0.00238/\rho)\ W/S$ for W/S wherever W/S occurs in the computations.

Correction to a different value of $dC_Y/d\beta$ may be readily made as follows: Compute ζ' for $dC_Y/d\beta=-0.28$, i. e., $y_v=-0.14$, from the relation $\zeta'=\frac{-0.313\sqrt{(W/S)C_L}}{T}$. Add to this value of ζ' the quantity $\frac{1}{4}(0.28+dC_Y/d\beta)$ to obtain the corrected value of ζ'. Calculate the corrected value of $T/\sqrt{W/S}$ using the corrected value of ζ'.

In cases where a large number of estimates are to be made for a given pair of values of W/S and b, it will sometimes be convenient to convert the charts to read directly in terms of $-dC_l/d\beta$, $dC_n/d\beta$, P, and T. This conversion can readily be accomplished without redrawing the chart by changing the constants. Figure 26 represents figure 6 converted to read directly in the desired quantities for an airplane having $W/S=16$ pounds per square foot and $b=42$ feet.

Example of use of charts.—It is assumed that the lateral-stability characteristics throughout the normal-flight range are desired for a 5,000-pound airplane having a wing loading of 16 pounds per square foot and a span of 42 feet. Values of $dC_n/d\psi$ and $dC_l/d\psi$ are available from wind-tunnel tests. Values of $dC_n/d\frac{rb}{2V}$ must be estimated. The airplane is a modern type with a fairly high top speed and is equipped with split flaps. Flaps were considered to be down at $C_L=2.0$ but up at $C_L=0.2$, 0.8, and 1.4.

The stability characteristics will be estimated for each of the C_L values of 0.2, 0.8, 1.4, and 2.0. Values of $-\mu dC_l/d\beta$ and $\mu dC_n/d\beta$ are determined at each value of C_L from the relationships

$$-\mu dC_l/d\beta=\frac{13.1\times16}{42}\times57.3\times(dC_l/d\psi)$$

and

$$\mu dC_n/d\beta=\frac{13.1\times16}{42}\times57.3\times(-dC_n/d\psi)$$

Values of $dC_n/d\frac{rb}{2V}$ are determined from the relationship

$$dC_n/d\frac{rb}{2V}=-\frac{C_{D_w}}{3}-2\frac{l^2}{b^2}\frac{S_t}{S}\frac{dC_{L_t}}{d\beta}$$

where C_{D_w} is taken from wind-tunnel tests of a similar wing, l/b and S_t/S are dimensional characteristics of the airplane, and $dC_{L_t}/d\beta$ is estimated using the relationship

$$\frac{dC_{L_t}}{d\beta}=\frac{5.5}{1+\frac{2}{b_t^2/S_t}}$$

where b_t is the height of the vertical tail surface. The values of C_{D_w}, $dC_n/d\frac{rb}{2V}$, $-\mu dC_l/d\beta$, and $\mu dC_n/d\beta$ are as follows:

C_L	C_{D_w}	$dC_n/d\frac{rb}{2V}$	$-\mu dC_l/d\beta$	$\mu dC_n/d\beta$
0.2	0.008	−0.051	0.20	0.180
.8	.025	−.057	.25	.180
1.4	.070	−.072	.45	.165
2.0	.400	−.182	.55	.130

From the various charts, values of T and P are determined, interpolations and extrapolations being made where necessary. The values of the stability characteristics at each value of C_L follow.

C_L	T (sec.)	P (sec.)	Divergence
0.2	2.1	3.0	None.
.8	3.1	5.5	Spiral.
1.4	3.1	5.8	Do.
2.0	[1] 2.2	6.4	None.

[1] Correction for the increase in $dC_Y/d\beta$ due to the high drag gives the corrected value of T as 1.3 seconds.

EFFECT OF THE GOVERNING FACTORS ON THE STABILITY CHARACTERISTICS

AERODYNAMIC FACTORS

Lateral force due to sideslip.—The lateral force due to sideslip is small, in general, but beneficial in its effect upon stability characteristics. As appears in equation (18), $dC_Y/d\beta$ adds directly to the damping coefficient, $\Delta\zeta'=\frac{1}{4}\Delta dC_Y/d\beta$. For the value of $dC_Y/d\beta=-0.28$, $\Delta\zeta'=-0.07$, which is sufficient to damp the lateral oscillation to one-half amplitude in 8 seconds for an airplane with a wing loading of 16 flying at 176 miles per hour. The effects of $dC_Y/d\beta$ on the period and on the tendency to diverge are negligible.

Rolling moment due to sideslip.—The rate of change of rolling-moment coefficient with sideslip plays a great part in determining the stability characteristics, as is apparent from a glance at the charts of figures 4 to 25. It is necessary for stability that $dC_l/d\beta$ be negative; the term $-dC_l/d\beta$ will be used, as in the charts, for simplicity in discussion.

Increasing $-dC_l/d\beta$ increases the range of values of $dC_n/d\beta$ within which there is no divergence, there being less likelihood of either spiral divergence or directional divergence as $-dC_l/d\beta$ is increased. Increasing $-dC_l/d\beta$ increases the time for an oscillation to damp and shortens the period. These effects are sufficiently small to be of no practical importance at high speeds but are appreciable at low speeds.

Yawing moment due to sideslip.—From the considerations of tendency toward divergence the value of $dC_n/d\beta$ should be small and positive. Too large a positive value of $dC_n/d\beta$ results in spiral divergence. Too large a negative value will lead to undamped lateral oscillations as indicated by the curve of $T/\sqrt{(W/S)}=\infty$ (oscillatory divergence) or to directional divergence. The range of permissible values of $dC_n/d\beta$ is quite narrow for small values of $-dC_l/d\beta$ and $-dC_n/d\frac{rb}{2V}$.

Increasing $dC_n/d\beta$ increases the damping and shortens the period of the lateral oscillations. The effect upon the period is very pronounced, particularly at small values of the lift coefficient corresponding to cruising and high speeds, as is especially apparent in equation (23) where

$$P=\sqrt{\frac{0.14b}{(-dC_l/d\beta)+\frac{3.2}{C_L}dC_n/d\beta}}$$

It appears that for $C_L=0.2$ the effect of $dC_n/d\beta$ upon the period has 16 times the effect of $-dC_l/d\beta$. The effect upon T is less pronounced. It is of interest to note that the theory indicates stability with $dC_n/d\beta$ zero or slightly negative if $dC_Y/d\beta$ and $dC_n/d\frac{rb}{2V}$ are moderate or large.

Rolling moment due to rolling.—Differences in the value of $dC_l/d\frac{pb}{2V}$ of the order of those likely to exist between conventional airplanes in the normal-flight range have but slight effect upon the tendency toward divergence or the oscillatory characteristics. This fact tends to justify the use of an average value for this factor in equations (20) and (23) and in the charts. The small effects occurring are such that increasing $dC_l/d\frac{pb}{2V}$ decreases the time required to damp, in general, and increases the period.

Near the stall $dC_l/d\frac{pb}{2V}$ changes sign and tends to result in violent instability. This report does not deal with stability near the stall, which is amply discussed in references 14, 15, and 16.

Yawing moment due to rolling.—As is the case for $dC_l/d\frac{pb}{2V}$, differences in $dC_n/d\frac{pb}{2V}$ likely to exist in practice have comparatively slight effect upon the stability characteristics below the stall. Increasing $dC_n/d\frac{pb}{2V}$ may either increase or decrease T, depending upon the magnitudes of other quantities, and increases P slightly. Here again the selection of an average value for this factor seems justified. Near the stall $dC_n/d\frac{pb}{2V}$ changes sign and becomes an important factor in producing instability.

Rolling moment due to yawing.—The rolling moment due to yawing is chiefly of importance in connection with the likelihood of spiral divergence. Increasing $dC_l/d\frac{rb}{2V}$ decreases the range of values of $dC_n/d\beta$ for which spiral convergence exists for a given set of values of $-dC_l/d\beta$ and $dC_n/d\frac{rb}{2V}$. Increasing $dC_l/d\frac{rb}{2V}$ generally decreases T but has no noticeable effect upon P or the likelihood of directional divergence.

Yawing moment due to yawing.—Increasing $dC_n/d\frac{rb}{2V}$ increases the permissible range of values of $dC_n/d\beta$ for spiral convergence and decreases the time required for the oscillation to damp to one-half amplitude. It is apparent from equations (20) and (13) and from the charts that an accurate knowledge of $dC_n/d\frac{rb}{2V}$ is essential to accurate calculations of T and of the limiting values of $dC_n/d\beta$ within which spiral convergence exists. On the other hand, $dC_n/d\frac{rb}{2V}$ has only a very slight effect upon the directional convergence or upon the period of the oscillations.

MASS FACTORS

Relative density of airplane to air.—The relative density μ has no effect upon the likelihood of either spiral or directional convergence. Its effect upon the period and damping of the lateral oscillation can best be understood by considering the separate effects of the factors which determine μ, namely W/S, b, and ρ. Since a decrease in ρ has precisely the same effect as an increase in W/S, the effects of altitude are the same as the effects of increasing the wing loading and will therefore not be discussed separately.

The effect of wing loading upon the time required to damp the oscillation to one-half amplitude can best be deduced from equations (16) and (20). From equation (20) it appears that if, for the case at hand $1.2\frac{dC_n}{d\beta}\left(-\frac{dC_n}{d\frac{rb}{2V}}\right)$ is greater than 0.3 $(-dC_l/d\beta)$, then increasing W/S (since μ is proportional to W/S) will make ζ' greater in the negative sense. This will be the case only for very small values of $-dC_l/d\beta$. In general, therefore, increasing W/S will increase T both by decreasing ζ' and by increasing the numerator in the relationship

$$T=\frac{-0.313\sqrt{(W/S)C_L}}{\zeta'}$$

On the other hand, wing loading has no appreciable effect upon the period, at a given C_L, as is apparent from equation (23).

From the charts it appears that, since increasing b decreases $\mu dC_n/d\beta$ and $-\mu dC_l/d\beta$, increasing the span will decrease the time required to damp the oscillations over most of the range of values of the parameters. As pointed out in the preceding paragraph, the effect depends upon the relative magnitudes of $dC_n/d\beta$, $dC_n/d\frac{rb}{2V}$, and $-dC_l/d\beta$. Only in the case of very small values of $-dC_l/d\beta$, will increasing the span increase T. For practical purposes the period of the lateral oscillation is proportional to the square root of the span, as is shown by equation (23).

Ratio of wing span to radius of gyration about X axis.—In the discussion of the effects of changing b it was assumed that the ratios b/k_X and b/k_Z were kept constant. The effects of changing these ratios can be most readily explained on the basis of keeping b constant.

The value of k_X has no effect upon either spiral or directional convergence. Although not readily apparent in equations (18) and (22), increasing k_X results in small increases in T and P. There is, however, no justification for extensive labor to determine k_X accurately in the absence of accurate data on all the aerodynamic factors.

Ratio of wing span to radius of gyration about Z axis.—The effects of increasing k_Z are similar to the effects of increasing k_X. It has a slight but unimportant effect upon directional convergence. Its effect upon the period is greater than the effect of increasing k_X, but not great enough to be of practical importance in most cases.

GENERAL COMMENTS

The present state of knowledge does not justify positive assertions as to the desirability of any given set of stability characteristics. Very little has been done to determine quantitatively the stability characteristics that result in the most satisfactory riding and handling characteristics. Such research (reference 17) has given more or less negative results, at least with respect to the period and the damping of oscillations. It is definitely known, however, that very great instability, such as that at the stall, and very great stability are both undesirable. There is strong reason to believe that any tendency to diverge is undesirable but that, if such a tendency is of small magnitude, it will not seriously inconvenience the pilot.

When the foregoing facts are taken into consideration, it seems desirable that for airplanes designed for most purposes, excepting machines intended as pursuits, fighters, or for acrobatics, there should be no tendency to diverge, oscillations should be moderately to heavily damped, and the period of the oscillations should be as long as practicable. It is believed that such characteristics will require a minimum of effort from the pilot and will result in a maximum of passenger comfort.

Reference to the charts of figures 4 to 25 reveals that these characteristics can be attained only by making $dC_n/d\beta$ small while keeping $dC_n/d\frac{rb}{2V}$ and $dC_Y/d\beta$ large. Some additional advantage is gained by keeping $-dC_l/d\beta$ small, particularly at high angles of attack. Probably the best value of $-dC_l/d\beta$ is from small to moderate, the moderate values giving more pronounced spiral and directional convergence. The best method of keeping $dC_n/d\beta$ small while retaining large values of $dC_n/d\frac{rb}{2V}$ and $dC_Y/d\beta$ appears to be the use of a fuselage giving an unstable yawing moment of rather large magnitude. The unstabilizing effect of the fuselage depends on its length, breadth, the distance of the center of gravity from the nose, and on the shape. The shape of the fuselage, and possibly interference effects, play an important part, which can be determined accurately only by wind-tunnel tests. A large value of l tends to make $dC_n/d\frac{rb}{2V}$ large and a large, deep fuselage tends to give a large value of $dC_Y/d\beta$. The dihedral of the wings can be adjusted to bring $dC_l/d\beta$ to the desired value but here again, with present knowledge, it is necessary to make wind-tunnel tests.

Although control is outside the intended scope of this paper, it should be pointed out that appearance of instability may, under certain circumstances, be brought about by the influence of the controls. The two most common instances are that of directional divergence arising out of an attempt to hold the wings level with conventional ailerons, the rudder being held neutral; and that of increasing or poorly damped oscillations arising out of operation of the rudder in improper phase relationship to the change in attitude of the airplane. The directional divergence is caused by the adverse yaw of the ailerons and can be avoided by reducing the adverse yaw, by increasing $dC_n/d\beta$, or by holding the ailerons neutral and allowing the airplane to roll. The increasing oscillations are most likely to occur when the natural period of the airplane is short and when the rudder is operated in such a manner as to prevent yawing. They can be avoided by holding the rudder neutral or by operating it in such a manner as to produce sideslip opposing the roll, i. e., by trying to hold the wings level rather than by trying to prevent yawing.

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SUGGESTIONS FOR FUTURE STUDY

A systematic correlation of stability characteristics with riding and handling qualities is needed. It is possible that these qualities are more directly related to certain of the governing factors than to the tendency to diverge or to the characteristics of the oscillations; investigations should be conducted with this possibility in mind.

There is need for adequate comparison between computed values and measured values of stability characteristics as a check upon the accuracy and validity of the mathematical treatment.

At present some of the aerodynamic governing factors cannot be estimated with assurance. A great deal of systematic study will be necessary to provide sufficient data for the formulation of satisfactory empirical constants to be used in estimating these factors.

In this report no consideration has been given to the effect of power on the lateral stability. This problem should be the subject of a study sufficiently thorough to reveal the effects of power on the stability derivatives and upon the mathematical treatment necessary to estimate the stability characteristics.

More satisfactory means of measuring the separate aerodynamic factors and also the final stability characteristics of models are necessary for rapid progress.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *November 17, 1936.*

APPENDIX I

DERIVATION OF FORMULAS

The theory of small oscillations.—The theory of small oscillations was first applied by Bryan to the dynamics of mechanical flight (reference 10). On the assumption that the direction and magnitude of changes in the aerodynamic characteristics due to changes in motion from the steady-flight condition are known, equations of motion in unsteady flight are written for the case of small deviations from the steady condition, one equation for each of the degrees of freedom of the motion. Simultaneous solution of the equations gives values that describe the motion of the airplane after a disturbance.

Assumptions in the application of the theory.—In the application of the theory of small oscillations to quantitative estimations of stability characteristics, a number of assumptions are necessary in order that the mathematics may not be too involved and the computations too extensive for practical applications. The primary assumptions are as follows:

(*a*) The combined aerodynamic effect of two or more components of motion is assumed equal to the algebraic sum of the separate effects of the individual components.

(*b*) The changes in aerodynamic forces and moments due to a deviation are assumed proportional to the deviation, i. e., the slopes $dC_l/d\beta$, $dC_l/d\frac{pb}{2V}$, etc., are assumed to be constants.

(*c*) The lateral motion involving p, q, and r is assumed to be independent of the longitudinal motion, i. e., the machine is assumed to be symmetrical.

(*d*) Secondary effects such as those involving the products of two or more small quantities are neglected.

(*e*) The values of the aerodynamic factors are assumed to be unaffected by the linear and angular accelerations.

Equations of lateral motion.—The equations of lateral motion will be written for the axes shown in figure 1 using the symbols and notation given in appendix III and on the report covers. The X axis is taken in the direction of the relative wind during the steady-flight condition. The axes are assumed fixed in the airplane. During steady flight,

$$\begin{aligned} Y&=L=N=0\\ v&=p=r=0\\ u&=V \end{aligned}$$

After a disturbance,

$$\left.\begin{aligned} &v\frac{dY}{dv}+p\frac{dY}{dp}+r\frac{dY}{dr}+W\sin\phi\cos\gamma\\ &\qquad+W\sin\psi\sin\gamma=m\frac{dv}{dt}+mru\\ &v\frac{dL}{dv}+p\frac{dL}{dp}+r\frac{dL}{dr}=mk_X^2\frac{dp}{dt}\\ &v\frac{dN}{dv}+p\frac{dN}{dp}+r\frac{dN}{dr}=mk_Z^2\frac{dr}{dt} \end{aligned}\right\}\tag{24}$$

$$\left.\begin{aligned} p&=d\phi/dt\\ r&=d\psi/dt \end{aligned}\right\}\tag{25}$$

It is assumed in these equations that the principal axes of inertia are coincident with the reference axes, which is not true in the general case. A number of supplementary calculations made as part of the study leading up to this report have indicated, however, that to neglect the angularity of the principal axes to the reference axes will not introduce serious error in the normal-flight range and will give slightly conservative results. Consequently, the terms including the product of inertia were omitted to make the equations as simple as possible.

Since dY/dp and dY/dr are small, they are generally neglected. For the small deviations considered, u may be taken equal to the steady-flight velocity V and the sines of the angles of roll and yaw may be replaced by the angles themselves. Since in power-off flight the lift is equal to $W\cos\gamma$ and the lift times the tangent of the angle of glide is equal to $W\sin\gamma$, the first of the foregoing equations will be rewritten,

$$v\frac{dY}{dv}+\phi\times(\text{lift})+\psi\times(\text{lift})\times\tan\gamma=m\frac{dv}{dt}+mrV$$

The equations of equilibrium finally become,

$$\left.\begin{aligned} &v\frac{dY}{dv}-m\frac{dv}{dt}+\phi(\text{lift})+\psi(\text{lift})\tan\gamma-\frac{d\psi}{dt}mV=0\\ &v\frac{dL}{dv}+\frac{d\phi}{dt}\frac{dL}{dp}-\frac{d^2\phi}{dt^2}mk_X^2+\frac{d\psi}{dt}\frac{dL}{dr}=0\\ &v\frac{dN}{dv}+\frac{d\phi}{dt}\frac{dN}{dp}+\frac{d\psi}{dt}\frac{dN}{dr}-\frac{d^2\psi}{dt^2}mk_Z^2=0 \end{aligned}\right\}\tag{26}$$

Replacing

$$\frac{dY}{dv}\text{ by }\frac{1}{2}\rho V^2S\frac{dC_Y}{dv}$$

$$(\text{Lift})\text{ by }\frac{1}{2}\rho V^2S\,C_L$$

$$\frac{dL}{dv} \text{ by } \frac{1}{2}\rho V^2 S\, b\frac{dC_l}{dv}, \text{ etc.}$$

$$\frac{dN}{dv} \text{ by } \frac{1}{2}\rho V^2 S\, b\frac{dC_n}{dv}, \text{ etc.}$$

$$\frac{m}{\rho Sb} \text{ by } \mu$$

$$\frac{m}{\rho SV} \text{ by } \tau$$

$$\frac{dC_Y}{dv} \text{ by } \frac{1}{V}\frac{dC_Y}{d\beta}$$

Writing in determinant form and simplifying gives,

$$\begin{vmatrix} v\, y_v - \tau\frac{dv}{dt} & \frac{C_L}{2}\phi & \frac{C_L}{2}(\tan\gamma)\,\psi - \tau\frac{d\psi}{dt} \\ v\,\mu l_v & l_p\tau\frac{d\phi}{dt} - \tau^2\frac{d^2\phi}{dt^2} & l_r\tau\frac{d\psi}{dt} \\ v\,\mu n_v & n_p\tau\frac{d\phi}{dt} & n_r\tau\frac{d\psi}{dt} - \tau^2\frac{d^2\psi}{dt^2} \end{vmatrix} = 0 \qquad (27)$$

where $y_v = \frac{1}{2}\frac{dC_Y}{d\beta}$

$$l_v = \frac{1}{2}\left(\frac{b}{k_X}\right)^2\frac{dC_l}{d\beta}$$

$$n_v = \frac{1}{2}\left(\frac{b}{k_Z}\right)^2\frac{dC_n}{d\beta}$$

$$l_p = \frac{1}{4}\left(\frac{b}{k_X}\right)^2\frac{dC_l}{d\frac{pb}{2V}}$$

$$n_p = \frac{1}{4}\left(\frac{b}{k_Z}\right)^2\frac{dC_n}{d\frac{pb}{2V}}$$

$$l_r = \frac{1}{4}\left(\frac{b}{k_X}\right)^2\frac{dC_l}{d\frac{rb}{2V}}$$

$$n_r = \frac{1}{4}\left(\frac{b}{k_Z}\right)^2\frac{dC_n}{d\frac{rb}{2V}}$$

Substituting $v_0\, e^{\lambda t}$ for v, $\phi_0\, e^{\lambda t}$ for ϕ, etc., and simplifying give

$$\begin{vmatrix} y_v - \tau\lambda & \frac{C_L}{2} & \frac{C_L}{2}\tan\gamma - \tau\lambda \\ \mu l_v & l_p\tau\lambda - \tau^2\lambda^2 & l_r\tau\lambda \\ \mu n_v & n_p\tau\lambda & n_r\tau\lambda - \tau^2\lambda^2 \end{vmatrix} = 0 \qquad (28)$$

which can be expressed as

$$A(\lambda')^5 + B(\lambda')^4 + C(\lambda')^3 + D(\lambda')^2 + E(\lambda') = 0 \qquad (29)$$

where,

$$\lambda' = \tau\lambda$$

$$A = 1$$

$$B = -y_v - n_r - l_p$$

$$C = l_p n_r - l_r n_p + y_v(l_p + n_r) + \mu n_v$$

$$D = y_v(l_r n_p - l_p n_r) + \mu l_v\left(n_p - \frac{C_L}{2}\right) - \mu n_v\left(\frac{C_L}{2}\tan\gamma + l_p\right)$$

$$E = \mu\frac{C_L}{2}(l_v n_r - l_r n_v) + \mu\frac{C_L}{2}\tan\gamma\,(l_p n_v - l_v n_p)$$

The solution $\lambda' = 0$ is readily apparent in equation (29) and results from the fact that the airplane has no inherent tendency to return to any fixed compass course, being, as the solution shows, neutrally stable in that respect. This solution, $\lambda' = 0$, is generally neglected and the lateral-stability characteristics are considered as those given by the biquadratic

$$A(\lambda')^4 + B(\lambda')^3 + C(\lambda')^2 + D(\lambda') + E = 0 \qquad (30)$$

The deviation of each one of the components of the lateral motion varies with time according to the relation,

$$v,\ p, \text{ or } r = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3 e^{\lambda_3 t} + C_4 e^{\lambda_4 t}$$

where λ_1, λ_2, λ_3, and λ_4 are the four roots of the biquadratic. In lateral motion the constants B, C, D, and E are generally such that there are one pair of real roots and one pair of conjugate complex roots indicating motion of the type,

$$v,\ p, \text{ or } r = C_5 e^{\zeta t}(\cos\psi t - C_6) + C_3 e^{\lambda_3 t} + C_4 e^{\lambda_4 t}$$

where ζ and ψ are the real and imaginary parts, respectively, of the conjugate roots. This motion represents an oscillation superimposed upon two rates of convergence (or divergence). It is evident that for stability ζ, λ_3, and λ_4 must be real and negative so that the values of v, p, and r will reduce to zero. In order that the real parts of the roots shall all be negative it is necessary and sufficient that B, C, D, E, and $(BCD - D^2 - B^2E)$ each be positive. In order to determine the rates of convergence and the damping and period of the oscillation, it is necessary to solve the biquadratic. A convenient semigraphical method of solving stability biquadratics was pointed out in reference 1 and is described in detail in appendix II.

APPENDIX II

SOLUTION OF STABILITY BIQUADRATIC

Semigraphical method of solving biquadratics.—The biquadratic

$$\lambda^4+B\lambda^3+C\lambda^2+D\lambda+E=0$$

can be expressed as

$$(\lambda^2+a_1\lambda+b_1)(\lambda^2+a_2\lambda+b_2)=0 \tag{31}$$

from which

$$\left.\begin{aligned}\lambda&=-\frac{a_1}{2}\pm\sqrt{\left(\frac{a_1}{2}\right)^2-b_1}\\ \lambda&=-\frac{a_2}{2}\pm\sqrt{\left(\frac{a_2}{2}\right)^2-b_2}\end{aligned}\right\} \tag{32}$$

are roots of the general equation. It appears that

$$\left.\begin{aligned}B&=a_1+a_2\\ C&=a_1a_2+b_1+b_2\\ D&=a_1b_2+a_2b_1\\ E&=b_1b_2\end{aligned}\right\} \tag{33}$$

Eliminating values of a_2 and b_2,

$$a_1=\frac{B}{2}\pm\sqrt{\left(\frac{B}{2}\right)^2-C+b_1+\frac{E}{b_1}} \tag{34}$$

and

$$a_1=\frac{b_1^2B-b_1D}{b_1^2-E} \tag{35}$$

Note that, if the minus sign in equation (34) is chosen for a_1, the plus sign will correspond to a_2.

Values of a_1 and b_1 that will satisfy these equations separately are plotted on charts having values of a as abscissas and values of b as ordinates. The intersection of the resulting curves represents values of a_1 and b_1 that satisfy both equations. There are two intersections in the general case, one corresponding to a_1 and b_1, the other to a_2 and b_2. Ordinarily it is more convenient to find one of the intersections by plotting and to solve for the remaining values by the use of equations (33).

Figure 27 gives a solution of a typical stability biquadratic and illustrates the use of this semigraphical method. For most cases time can be saved in locating the intersection by letting a_1 equal zero in equation (35), thus determining the intersection of the plot of equation (35) with the b axis. The resulting value of b_1 when substituted in equation (34) will give an approximate value for a_1. The final values can then be determined as accurately as desired by locating the point of intersection of the curves. This method has been applied to several hundred solutions of stability biquadratics in the course of the study leading up to this report and has been found to be very satisfactory, particularly so if systematic changes in factors are being studied.

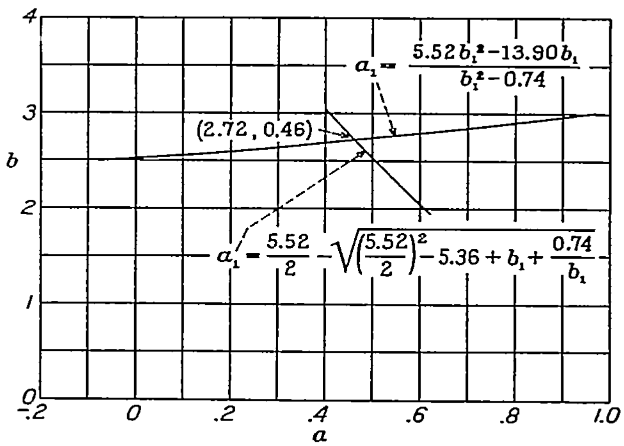

FIGURE 27.—A semigraphical method of solving stability biquadratics. Solution of biquadratic:

$$\lambda^4+5.52\lambda^3+5.36\lambda^2+13.90\lambda+0.74=(\lambda^2+a_1\lambda+b_1)(\lambda^2+a_2\lambda+b_2)=0$$

From intersection of curves

$$a_1=0.46;\ b_1=2.72$$

From equations (33)

$$a_2=5.52-0.46=5.06;\ b_2=0.74/2.72=0.027$$

From $\lambda^2+0.46\lambda+2.72=0$, $\lambda=-0.23\pm i\,1.6$
From $\lambda^2+5.06\lambda+0.027=0$, $\lambda=-5.05$
$\lambda=-0.01$

Approximate formulas for the damping and the period of the oscillation.—As was stated in the preceding paragraph, an approximate value of b_1 can be found by substituting $a_1=0$ in equation (35) giving

$$b_1=\frac{D}{B} \tag{36}$$

Substitution of this value of b_1 in equation (34) gives the approximate value for a_1 of

$$a_1=\frac{B}{2}\pm\sqrt{\left(\frac{B}{2}\right)^2-C+\frac{D}{B}+\frac{EB}{D}} \tag{37}$$

This latter equation can be further simplified without loss of accuracy by removing the radical and assuming $a_1^2=0$. The resulting equation for a_1 is

$$a_1=\frac{C}{B}-\frac{D}{B^2}-\frac{E}{D} \tag{38}$$

Since

$$\lambda=-\frac{a_1}{2}\pm\sqrt{\left(\frac{a_1}{2}\right)^2-b_1}$$

$$=\zeta'\pm i\psi' \tag{39}$$

$$\zeta'=-\frac{a_1}{2} \tag{40}$$

$$\psi'=\sqrt{-\left(\frac{a_1}{2}\right)^2+b_1}$$

Since $\left(\frac{a_1}{2}\right)^2$ is normally small compared with b_1, it is sufficiently accurate for practical purposes to put

$$\psi'=\sqrt{b_1} \tag{41}$$

From equations (38) and (36),

$$\zeta'=-\frac{1}{2}\left[\frac{C}{B}-\frac{D}{B^2}-\frac{E}{D}\right] \tag{42}$$

$$\psi'=\sqrt{\frac{D}{B}} \tag{43}$$

Supplementary study has indicated that further simplification can be had with but slight loss in accuracy by neglecting values of y_v in the expression for B and D and neglecting tan γ in the expression for D and E giving

$$\left.\begin{aligned}B&=-n_r-l_p\\C&=l_pn_r-l_rn_p+y_v(l_p+n_r)+\mu n_v\\D&=\mu l_v\left(n_p-\frac{C_L}{2}\right)-\mu n_vl_p\\E&=\mu\frac{C_L}{2}(l_rn_r-l_rn_v)\end{aligned}\right\} \tag{44}$$

Substituting these values in equations (42) and (43) and simplifying where possible gives

$$\zeta'=-\frac{1}{2}\left[-y_v+\frac{l_pn_r-l_rn_p}{(-n_r-l_p)}+\frac{\mu n_v(-n_r)}{(-n_r-l_p)^2}+\frac{C_Ll_rn_v}{2l_v\left(n_p-\frac{C_L}{2}\right)-2n_vl_p}\right]+\frac{1}{2}\left[\frac{\mu l_v\left(n_p-\frac{C_L}{2}\right)}{(-n_r-l_p)^2}+\frac{C_Ll_vn_r}{2l_v\left(n_p-\frac{C_L}{2}\right)-2n_vl_p}\right] \tag{45}$$

and

$$\psi'=\sqrt{\frac{\mu l_v\left(n_p-\frac{C_L}{2}\right)-\mu n_vl_p}{(-n_r-l_p)}} \tag{46}$$

The time in seconds to damp to one-half amplitude is given by

$$T=\frac{\log_e 0.5}{\zeta}=\frac{-0.693}{\zeta'}\tau$$

(since $\zeta'=\tau\zeta$).

Expressed in more convenient form, this equation becomes

$$T=\frac{-0.313\sqrt{\left(\frac{W}{S}\right)\frac{\rho_0}{\rho}C_L}}{\zeta'} \tag{47}$$

The period in seconds of the lateral oscillation is

$$P=\frac{2\pi}{\psi}=\frac{2\pi\tau}{\psi'}$$

or

$$P=\frac{2.83\sqrt{\left(\frac{W}{S}\right)\frac{\rho_0}{\rho}C_L}}{\psi'} \tag{48}$$

Approximate formulas for the convergence characteristics.—Since a_1 is small, a_2 is approximately equal to B.

Letting $a_2=B$

and $b_2=\frac{EB}{D}$

gives

$$\lambda=-\frac{B}{2}\pm\sqrt{\left(\frac{B}{2}\right)^2-\frac{EB}{D}} \tag{49}$$

as the solution for the pair of the roots of the stability biquadratics corresponding to the convergence characteristics. Since B is always positive and large in the normal-flight range, it appears that this equation represents two convergences if EB/D is positive and less than $(B/2)^2$, a heavily damped oscillation if EB/D is positive and greater than $(B/2)^2$, a divergence and a convergence if EB/D is negative and less than $(B/2)^2$, and two divergences if EB/D is negative and greater than $(B/2)^2$. Instability is therefore possible if either E or D becomes negative. For most cases E is small but may be either positive or negative and D is positive and large. These circumstances give the usual solution of (49) as a large negative root approximately equal to $-B=n_r+l_p$ and a small root approximately equal to $-E/D$.

In the usual case it is desired to know whether or not there will be a divergence rather than to know the rapidity of the convergence. For such a case it is sufficient to know that

$$D>0$$

and

$$E>0$$

By the use of the relationships of equations (44), these conditions are represented by

$$l_v\left(n_p-\frac{C_L}{2}\right)-n_vl_p>0 \tag{50}$$

and

$$l_vn_r>l_rn_v \tag{51}$$

These equations neglect the effects of y_v and tan γ, a procedure that is conservative for power-off flight.

APPENDIX III

SYMBOLS

X, Y, Z, axes of reference fixed in the airplane having the origin at the center of gravity, the X axis in the plane of symmetry and along the relative wind in steady flight, the Y axis perpendicular to the plane of symmetry, and the Z axis in the plane of symmetry and perpendicular to the X axis.

X, Y, Z, forces along the respective axes, X being positive when directed forward, Y positive when directed to the right, and Z positive when directed downward.

L, M, N, moments about the X, Y, and Z axes, respectively, L being positive when it tends to depress the right wing, M positive when it tends to depress the tail, and N positive when it tends to retard the right wing.

u, v, w, components of linear velocity of the airplane along the X, Y, and Z axes, respectively, having the same positive directions as the X, Y, and Z forces.

V, resultant velocity.

p, q, r, components of angular velocity about the X, Y, and Z axes, respectively, having the same positive directions as L, M, and N.

ϕ, θ, ψ, components of angular displacement from a given attitude about the X, Y, and Z axes, respectively.

α, angle between the relative wind on a plane parallel to the plane of symmetry and the wing chord, positive when corresponding to positive rotation θ of the airplane relative to the wind.

β, angle between the relative wind and a plane parallel to the plane of symmetry, equal to $\sin^{-1}\frac{v}{V}$; angle of sideslip in radians.

γ, angle of flight path to horizontal, positive in a climb.

Γ, dihedral angle, degrees.

$C_Y=\frac{Y}{qS}$, coefficient of lateral force.

$C_l=\frac{L}{qSb}$, coefficient of rolling moment.

$C_n=\frac{N}{qSb}$, coefficient of yawing moment.

C_{D_w}, coefficient of drag for the wing alone.

C_{L_p}, coefficient of force on projected side area.

C_{L_t}, coefficient of force on vertical-tail area.

S, wing area.

S_s, projected side area of fuselage.

S_t, vertical-tail area.

S_p, projected side area.

d, maximum depth of fuselage.

y, spanwise distance from plane of symmetry.

x_1, distance from fuselage nose to center of gravity.

l, distance from center of gravity to rudder hinge.

l_1, over-all length.

l_2, over-all length of fuselage.

ρ_0, mass density of air under standard conditions.

ρ, mass density of air under condition of flight.

t, subscript denoting vertical tail surfaces.

b_t, height of vertical tail.

η_t, tail efficiency.

z_p, the Z coordinate of the center of pressure of projected side area.

K_β, empirical factor for estimating $dC_n/d\beta$ for fuselage.

$\mu=\frac{m}{\rho Sb}$, relative density factor.

For standard atmosphere, $\mu=\frac{13.1(W/S)}{b}$.

$\tau=\frac{m}{\rho SV}=\mu\frac{b}{V}$, time conversion factor.

$y_v=\frac{1}{2}\frac{dC_Y}{d\beta}$, nondimensional derivative of lateral force due to sideslip.

$l_v=\frac{1}{2}\left(\frac{b}{k_X}\right)^2\frac{dC_l}{d\beta}$, nondimensional derivative of rolling moment due to sideslip.

$n_v=\frac{1}{2}\left(\frac{b}{k_Z}\right)^2\frac{dC_n}{d\beta}$, nondimensional derivative of yawing moment due to sideslip.

$l_p=\frac{1}{4}\left(\frac{b}{k_X}\right)^2\frac{dC_l}{d\frac{pb}{2V}}$, nondimensional derivative of rolling moment due to rolling.

$n_p=\frac{1}{4}\left(\frac{b}{k_Z}\right)^2\frac{dC_n}{d\frac{pb}{2V}}$, nondimensional derivative of yawing moment due to rolling.

$l_r=\frac{1}{4}\left(\frac{b}{k_X}\right)^2\frac{dC_l}{d\frac{rb}{2V}}$, nondimensional derivative of rolling moment due to yawing.

$n_r=\frac{1}{4}\left(\frac{b}{k_Z}\right)^2\frac{dC_n}{d\frac{rb}{2V}}$, nondimensional derivative of yawing moment due to yawing.

B, C, D, E, coefficients of stability biquadratic.

$\lambda=\frac{\lambda'}{\tau}=\zeta\pm i\psi=\frac{\zeta'}{\tau}\pm i\frac{\psi'}{\tau}$, root of stability equation.

T, time for oscillation to decrease to one-half amplitude, seconds.

$$T=\frac{-0.693}{\zeta'}\times\tau$$

$$=\frac{-0.313\sqrt{(W/S)C_L}}{\zeta'} \text{ (Standard atmosphere).}$$

P, period of oscillation, seconds.

$$P=\frac{2\pi\tau}{\psi'}$$

$$=\frac{2.83\sqrt{(W/S)C_L}}{\psi'} \text{ (Standard atmosphere).}$$

a, b, coefficients of stability quadratic.

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